

Active Particle Condensation by Nonreciprocal and Time-delayed Interactions

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We consider flocking of self-propelling agents in two dimensions, each of which communicates with its neighbors within a limited vision-cone (VC). Also, the communication occurs with some delay. The communication among the agents are modeled by Vicsek-rules. In this study we explore the effect of non-reciprocal interaction among the agents, induced by their VC, together with the delayed interactions on the dynamical pattern formation within the flock. We find that under these two influences and without any position-based attractive interactions or confining boundaries, the agents can spontaneously condense into ‘drops’. Though the agents are in motion within the drop, the drop as whole is virtually pinned in space. We also find that this novel state of the flock has a well defined order stabilized by the noise present in the system.

I. INTRODUCTION

The co-operative movement of *self-propelling* individuals or *agents*, ranging from cellular and colloidal scales [1–7] to the scales of the flocks of macroscopic living entities (e.g. birds, fishes, locusts etc.) [8–11] depends on their internal degrees of freedom that consume and dissipate energy to their local environment in far from equilibrium conditions. The collective dynamics of active systems often leads to features (e.g. dynamic pattern-formations and swarming) [12–18] which are not achievable in passive systems at equilibrium, or even in systems driven away from equilibrium by external fields. Recently their study has been contributing significantly to our current understanding of physics of living systems.

In many collections of living agents on the move the main concern is to maintain the speed and avoid collisions with other nearby agents. Both of these requirements are achievable to an extent by adopting a simple rule, such as ‘move as your neighbors do’ – that is, each agents attempts continuously to align its heading (direction of motion) to the average heading of its neighbors. Vicsek et al. [12] proposed a model of velocity alignment which incorporates this behavior. This model manifests some of the important characteristics of natural flocks – such as ordered collective motion (e.g. band formation) for high agent densities and low noise levels, and uncorrelated motion for low agent densities and high noises. Some recent reviews on this topic are [19, 20].

Two important features of inter-agent interaction are: (1) time delay and (2) reciprocity (or its absence). All the communications among agents in nature must have

some finite amount of delay. It is determined both by the speed of the signal used to communicate and the time taken by an agent to process the information regarding the communication. The communication and interaction among the agents are often non-reciprocal in the sense that the influence of, say agent i on agent j is in general different from that of the agent j on i . A simple example is human pedestrian motion. Each individual in a pedestrian crowd usually notices a person in front of him, but *is not noticed in turn by him*.

The model we have studied here incorporates both communication delay and non-reciprocity. Time delay is introduced in the dynamics of the agents by making agents ‘slow-to-notice’ the velocities of their neighbors by one time step (i.e., an agent uses neighbors’ velocities which are ‘out-of-date’ by one time-step to determine its next step). Non-reciprocity is introduced by assigning a limited (i.e. $\leq 2\pi$) angular range of interaction to every agent. This limited angular range is termed as vision cone (VC). As an example of how a vision cone can induce non-reciprocal interaction, one may consider a special case where, at time t , the agent j affects the motion of agent i because j is within the vision cone of i . But at the same time, it may happen that i is not in the vision cone of j , and thus i does not affect the dynamics of j . This is illustrated in Figure 1.

In this work these two important features of many natural flocks are considered together to investigate their interplay. It is by and large unexplored previously. Each of the features has been explored separately in recent studies. The role of time-delay has been explored by [21] where phototactic robots are found to form metastable clusters depending on their inherent sensory delay (delay to sense the optical signal) and the intensity of the signal. In another set of studies the effects of the delay of an individual agent of a flock to communicate with its neighbors are investigated [22–25]. It has been shown that in

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presence of a noise induced transition from translatory to rotatory motion of agents [26], the time-delayed communication among them can introduce further instabilities where the harmonically interacting agents can form dynamic clusters or swarms, with a high degree of polarity [22].

Reciprocity is the norm in the interactions among passive particles (atoms, molecules, colloidal particles etc.), though there is evidence of non-reciprocal, effective, physical interactions in out-of-equilibrium conditions. For example, when a system of particles is embedded in and interacts with each other via a moving, out-of-equilibrium medium, the effective interaction among the particles can be non-reciprocal. In particular, diffusiophoretic and optical forces within colloids [27–30], effective interaction among colloidal particles within a flowing solvent [31–33], shadow or wake mediated forces between particles within flowing plasma [34–36] are examples for non-reciprocal, physical interactions. But in such cases one can recover the reciprocity considering the medium and the particles together.

In contrast, interactions [37, 38] among the agents of a flock are often non-reciprocal, a fact that is barely appreciated in the much of the literature on active systems. The issue of non-reciprocity has been incidentally encountered (but not addressed specifically) in a few studies that explore the role of limited vision-cone of active, interacting agents in a flock. Angular restriction on the reorientation of an individual agent within a flock alone can facilitate ordering [39] and can also affect the thermodynamic character of the order-disorder transition occurs within the flocks [40]. It was also observed that angular restriction on interaction neighborhood in Vicsek model can reduce time required to reach ordered state from a disordered state [41]. Recently it has also been shown that VC can induce complex, self-assembled structures within the flock of self-propelling, memory-less agents, communicating among themselves by position-based, attractive interactions [42].

In contrast to the earlier studies, here we introduce the delay in communication among the agents and the angular restriction together, by adapting the Vicsek model. We report here emergence of an instability leading to absorbing states of the flock with frozen fluctuations [43–45]. In particular our study shows that agents with narrow vision cones and delayed responses spontaneously condense and confine themselves within small regions, eventually forming a few randomly positioned, dense clusters, which we call ‘drops’. Within such a drop, agents are in motion but the drop as whole is almost immobile, with only small fluctuations in the position of its center of mass. Also, the average angular momentum of the agents within the drop fluctuates about the zero, which means these drops do not have vortex-like dynamics either. The small size and immobility of the drop is sustained because individual agents perform a sequence of correlated, large-angle turn-arounds, effectively confining themselves to the drop. Importantly, we find that

the drops are stable only if there is a finite amount of noise present in the interaction among the agents (the drop disperses if the noise strength falls below a critical level). In the following sections we describe our model and report the results in detail.

II. MODEL

We add communication-delay among the agents together with non-reciprocity to a collection of Vicsek-agents. The non-reciprocity is taken care of by the VC, i.e. the interaction neighborhood of an agent which is not a circle centered around that agent (as it was in original Vicsek model), but a sector of this circle, as illustrated in Fig. 1. The neighborhood sector S_i has an opening angle of 2ϕ and is centered about the direction of velocity of the i^{th} particle. We shall call the half opening angle ϕ as the ‘view-angle’, which can vary from 0 to π . For $\phi = \pi$, without time-delay, this model reduces to the original Vicsek model.

At time t agents have positions $\mathbf{r}_i(t)$ and velocities $\mathbf{v}_i(t)$. Now we find the velocities at time $t + 1$ with

$$\hat{\mathbf{v}}_i(t+1) = \frac{\sum_{j \in S_i} \mathbf{v}_j(t)}{\left| \sum_{j \in S_i} \mathbf{v}_j(t) \right|}, \quad (1)$$

followed by

$$\mathbf{v}_i(t+1) = v_0 \mathcal{R}(\theta) \hat{\mathbf{v}}_i(t+1), \quad (2)$$

where the over-hat $\hat{\cdot}$ indicates a unit vector. And $|\dots|$ denotes the norm of the vector, $\mathcal{R}(\theta)$ is the rotation operator which rotates the vector it acts upon (i.e., $\hat{\mathbf{v}}_i(t+1)$) by an angle θ . The angle θ is a random variable uniformly distributed over the interval $[-\eta\pi, \eta\pi]$, where η is the level (i.e., amplitude) of the noise that can be varied from 0 to 1. v_0 is the constant speed of the agents. It should be noted here that the neighborhood S_i is as seen at time instant t .

Though we calculate $\mathbf{v}_i(t+1)$ but we update the positions with $\mathbf{v}_i(t)$, i.e.,

$$\mathbf{r}_i(t+1) = \mathbf{r}_i(t) + \mathbf{v}_i(t)\Delta t, \quad (3)$$

where $\Delta t = 1$. Thus positions are updated with velocities which lag by one time step. This introduces the delay in ‘response’ of an agent to the motion of its neighbors.

The degree of order in the collective motion of the particles is measured by a scalar order parameter $\psi(t)$ is defined as

$$\psi(t) = \frac{1}{Nv_0} \left| \sum_{i=1}^N \mathbf{v}_i(t) \right|. \quad (4)$$

In the perfectly ordered state when all the particles are moving in the same direction $\psi(t) = 1$, and in the completely disordered state when the directions of motion are

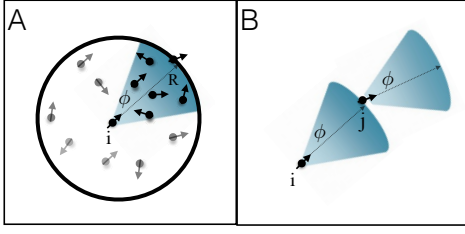


FIG. 1. (A) The neighborhood S_i of the i^{th} agent. The i^{th} agent is shown at the center of a circle of radius R and the neighborhood S_i is the blue sector of the circle. The black circles with black arrows as heading directions (*including the particle at the center of the circle*) indicate the agents lying within the neighborhood, and the gray circles with gray arrows as heading directions indicate agents outside it. The view-angle ϕ is the half opening angle of the neighborhood at the center. (B) An example of non-reciprocal configuration of agents i and j , where i interacts with j but not the other way round.

completely random $\psi(t) = 0$ (in the limit of $N \rightarrow \infty$). In this report we use the phrase ‘ordered state’ to mean the stationary state of the system for which $\psi(t) > 0$ in the limit of $N \rightarrow \infty$.

III. VC AND NON-RECIPROCITY

Here we will discuss how VC can induce non-reciprocity in the inter-agent interactions. The interaction between i^{th} and j^{th} agents, namely V_{ij} is non-reciprocal when $V_{ij} \neq V_{ji}$. For simplicity here we consider only two self-propelling agents, in zero noise limit, with directions of motion $\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2$ at time t . But the discussion here is applicable an arbitrary number of agents.

As described in Model section, the directions of motion of the agents can be updated at $(t+1)$ as,

$$\hat{\mathbf{v}}_1(t+1) = \frac{\hat{\mathbf{v}}_1(t) + \Theta(\phi - \delta_{12})\hat{\mathbf{v}}_2(t)}{1 + \Theta(\phi - \delta_{12})} \quad (5)$$

and

$$\hat{\mathbf{v}}_2(t+1) = \frac{\hat{\mathbf{v}}_2(t) + \Theta(\phi - \delta_{21})\hat{\mathbf{v}}_1(t)}{1 + \Theta(\phi - \delta_{21})} \quad (6)$$

Here $\delta_{12} = \angle(\hat{\mathbf{v}}_1, \hat{\mathbf{r}}_{12})$ and $\delta_{21} = \angle(\hat{\mathbf{v}}_2, \hat{\mathbf{r}}_{21})$. Here $0 \leq (\delta_{12}, \delta_{21}) \leq \pi$. Θ is the Heaviside step function i.e. $\Theta(x) = 1$ when $x \geq 0$ and $\Theta(x) = 0$ if $x < 0$ for any real x . If $\phi = \pi$, we recover Vicsek-like velocity alignment in zero noise limit.

Using $\Theta(x) = \frac{1}{2} + \frac{1}{2} \lim_{q \rightarrow \infty} \tanh(qx)$, equations [5,6] can be rewritten as

$$\hat{\mathbf{v}}_1(t+1) = \frac{\hat{\mathbf{v}}_1(t) + \frac{1}{2}(1 + \lim_{q \rightarrow \infty} \tanh(q\sigma_{12}))\hat{\mathbf{v}}_2(t)}{1 + \frac{1}{2}(1 + \lim_{q \rightarrow \infty} \tanh(q\sigma_{12}))} \quad (7)$$

and

$$\hat{\mathbf{v}}_2(t+1) = \frac{\hat{\mathbf{v}}_2(t) + \frac{1}{2}(1 + \lim_{q \rightarrow \infty} \tanh(q\sigma_{21}))\hat{\mathbf{v}}_1(t)}{1 + \frac{1}{2}(1 + \lim_{q \rightarrow \infty} \tanh(q\sigma_{21}))} \quad (8)$$

where $\sigma_{12} = \phi - \delta_{12}$ and $\sigma_{21} = \phi - \delta_{21}$. Here q is a positive parameter signifying the sharpness of the VC boundary. Note that if agent “2” is (not) within the VC of agent “1”, σ_{12} is positive (negative). Similarly, agent “1” is (not) within the VC of agent “2”, σ_{21} is positive (negative).

As $\tanh(-x) = -\tanh(x)$, for large q , $\tanh(\pm q\sigma_{12}) \simeq \pm(1 - 2\exp(-2q|\sigma_{12}|))$. Using this asymptotic expansion when agent “2” is inside the vision cone of “1”, Eq. 7 becomes,

$$\hat{\mathbf{v}}_1(t+1) \simeq \frac{1}{2}(\hat{\mathbf{v}}_1(t) + \hat{\mathbf{v}}_2(t)) + \frac{1}{4}(\hat{\mathbf{v}}_1(t) - \hat{\mathbf{v}}_2(t))e^{-q|\sigma_{12}|} \quad (9)$$

In Eq. 9 The first term is Vicsek-like aligning term, and the second term corresponds to the finite but large sharpness of VC boundary. It is straight forward to see that similar expression can be obtained for $\hat{\mathbf{v}}_2(t+1)$ from Eq. 8 using the asymptotic expansion, where σ_{12} will be replaced by σ_{21} as follows,

$$\hat{\mathbf{v}}_2(t+1) \simeq \frac{1}{2}(\hat{\mathbf{v}}_2(t) + \hat{\mathbf{v}}_1(t)) + \frac{1}{4}(\hat{\mathbf{v}}_2(t) - \hat{\mathbf{v}}_1(t))e^{-q|\sigma_{21}|} \quad (10)$$

In general δ_{12} is independent of δ_{21} , and therefore $\sigma_{12} \neq \sigma_{21}$. Thus, though the first term of Eqs. 9,10, i.e. the Vicsek-like term is symmetric under the exchange of the agents, the second term is not. And this is the root cause for non-reciprocity. Eq. 9,10 can be made concise for any time-step Δt as,

$$\begin{aligned} \frac{\Delta \hat{\mathbf{v}}_i}{\Delta t} &= \frac{(\hat{\mathbf{v}}_j - \hat{\mathbf{v}}_i)}{2\Delta t} \left(1 - \frac{1}{2}e^{-q|\sigma_{ij}|}\right) \\ &= \mathbf{F}_{ij} \left(1 - \frac{1}{2}e^{-q|\sigma_{ij}|}\right) \end{aligned} \quad (11)$$

where $(i, j) \in (1, 2)$ and $i \neq j$. From above dynamics, $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$ can be thought of as a reciprocal force acting between i^{th} and j^{th} agents and the non-reciprocity comes from the fact that $\sigma_{ij} \neq \sigma_{ji}$. Here we like to point out that the non-reciprocity is not a consequence of the finite sharpness of VC boundary. At $q \rightarrow \infty$ limit, the non-reciprocal part of the interaction will have significant contribution to the dynamics when $|\sigma_{ij}|$ is infinitesimally small so that the product $q|\sigma_{ij}|$ remains finite. But in $q|\sigma_{ij}| \rightarrow \infty$ limit we recover the reciprocal Vicsek-like interaction. The interaction above in Eq. 12 is similar to the one to simulate *passive non-reciprocal system* in [46].

When agent “2” is not inside the VC of “1”, Eq. 7 becomes,

$$\hat{\mathbf{v}}_1(t+1) \simeq \hat{\mathbf{v}}_1(t) + (\hat{\mathbf{v}}_2(t) - \hat{\mathbf{v}}_1(t))e^{-q|\sigma_{12}|} \quad (12)$$

Similar expressions can also be obtained for $\hat{\mathbf{v}}_2(t+1)$ from Eq. 8 where instead σ_{12} , we will have σ_{21} . As $\sigma_{ij} \neq \sigma_{ji}$, in this case also the system is in general non-reciprocal. In the limit of $(q|\sigma_{ij}|, q|\sigma_{ji}|) \rightarrow \infty$ the system becomes non-interacting and therefore trivially reciprocal.

IV. SIMULATION DETAILS

The simulations were carried out in two-dimensional ($2d$) square box of size L . At time $T = 0$, ' N ' agents are placed randomly and uniformly in the square box of size L . The positions of the agents are denoted by $\mathbf{r}_i; i = 1, 2 \dots N$. The mean agent density is given by $\rho = N/L^2$. For this study we carried out simulations with system sizes $N = 144, 256, 400$ and 576 . Directions to the agents ($\theta_i; i = 1, 2 \dots N$) are assigned randomly and uniformly in the range $[-\pi$ to $+\pi]$. Here θ_i is an angle between the velocity vector \mathbf{v}_i of the i^{th} agent with fixed frame of reference (in our case, the edge of the box). The speed of all the agents are same, constant and is denoted by v_0 . Then at each simulation time step the positions $\mathbf{r}_i(t)$ and velocities $\mathbf{v}_i(t)$ of all the agents are updated simultaneously according to the model described in the section II. The periodic boundary conditions are applied in the simulations.

The data presented in the Fig 2, 9 and 10 are computed over 20 configurations and each configuration is a simulation with total run time of 10^5 simulation time steps. We calculated average of the quantities after system has reached to the steady state. For that purpose we discard the initial data for 2×10^4 time steps in order to allow system to reach to the steady state in each configuration. The quantities are averaged over time as well as over multiple configurations. Throughout the study the following parameters are held constant : radius of interaction $R = 1.0$, density of agents $\rho = 1.0$ particle per unit area, and the speed of particles $v_0 = 0.5$ units per time step.

In the Fig. 2 we see one remarkable feature, which is the main concern of this paper. In the region of view-angle $\phi = 0.20\pi$ to $\phi = 0.28\pi$ the order parameter $\psi(t)$ is lower than the value of order parameter $\psi(t)$ for the disordered state (i.e. at $\phi = 0$). The non zero value of the order parameter $\psi(t)$ at $\phi = 0$ is due to finite size of the system. In this region the agents form a 'drop like state'. To analyze the 'drop like state' we choose mid point of the region. i.e. we choose view-angle $\phi = 0.24\pi$ and is fixed for the data presented in the Fig. 4, 6, 7. For these figures the system size is $N = 576$ agents and only a single configuration is used. The other parameters are constant as described above.

V. RESULTS AND DISCUSSION

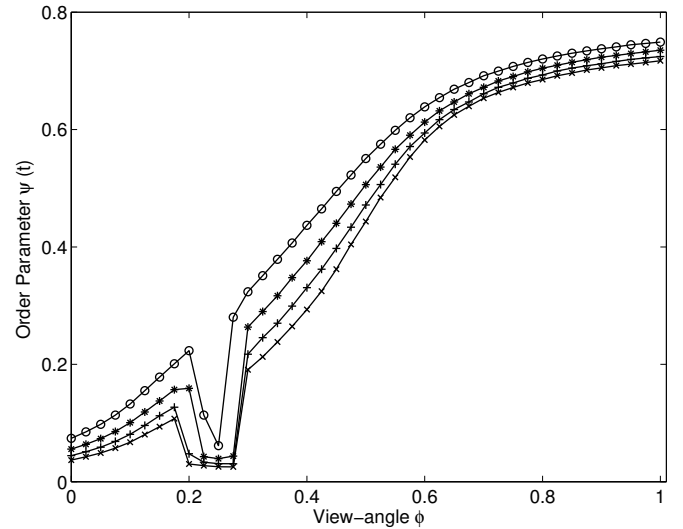


FIG. 2. Plot of order parameter $\psi(t)$ vs view-angle ϕ (in units of π). The circle, star, plus sign and cross corresponds to system size $N = 144, 256, 400, 576$ respectively. The parameters are radius of interaction $R = 1$, density $\rho = 1$, noise $\eta = 0.3\pi$. Each point is calculated over 20 configurations and for each configuration the averaging of $\psi(t)$ is carried out from $T = 2 \times 10^4$ to $T = 10^5$ simulation time steps.

Figure 2 shows steady-state average (over time and multiple configurations) of Vicsek order parameter $\psi(t)$ as a function of the view-angle ϕ for four different system sizes. Here we see the most remarkable anomalous behavior of the system - the order parameter $\psi(t)$ dips to a value close to zero around $\phi = 0.28\pi$, and then again recovers to slightly higher value at lower ϕ values. As the system size increases, the range of ϕ over which the 'dip' exists, widens. Within this anomalous range of ϕ ($\approx 0.20\pi$ to 0.28π) the value of the order parameter $\psi(t)$ is slightly lower than what is expected for a completely disordered state (which yields a small non zero value of the order of 10^{-2} due to finite system size), and moreover, even the *fluctuations* in this value are anomalously low. As we shall describe below, in this range of ϕ , the system is indeed not in a disordered state, but in a remarkable new state where the agents spontaneously confine themselves in a small, almost immobile cluster or *drop*. Vicsek order parameter $\psi(t)$ for this drop is as close to zero as it is for completely disordered state of the agents. Therefore $\psi(t)$ is inadequate to capture the difference between this drop state and completely disordered state.

We first describe the emergence this new kind of condensed state with a sequence of snap-shots in time, shown in figure 3. We choose the parameters in the 'dip region' of the figure 2, as described in the last paragraph of the previous section. The initial agent positions are homogeneously distributed in the simulation box, and velocities are random. As the time progresses we observe transient

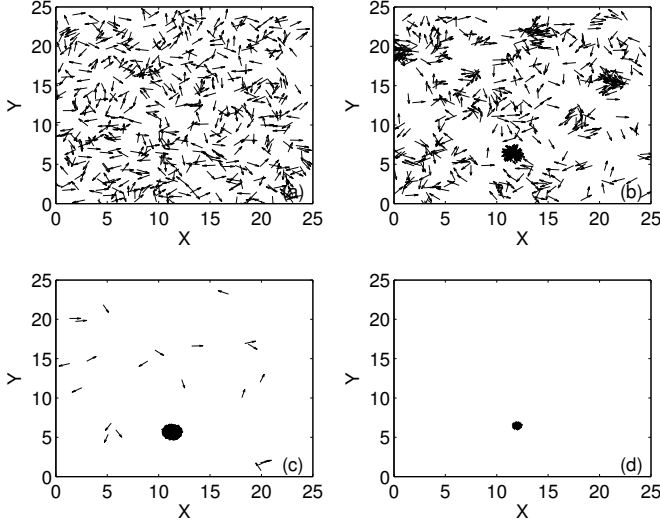


FIG. 3. Snapshot of the system at various instances. The number of particles are $N = 576$. Arrow indicates the direction of the motion of a particle. Snapshot corresponds to time for (A) 0 steps, the initial configuration (B) 168 steps, onset of drop formation (C) 1802 steps, growth of the drop (D) 4500 steps, steady state of the drop. The parameters are $N = 576, R = 1, \rho = 1, \phi = 0.24\pi, \eta = 0.3\pi$.

nucleation of of agents at many points within the the box. With time we observe emergence of one stable nucleus, which acts as a sink for the agents that pass close-by. Once a drifting agent gets within one step-length of the sink it is irreversibly captured, and this process continues until all the agents condense into a single drop of the size roughly equal to one step-length. This whole process can be clearly seen in the video1.mp4[47]. For the system size used for this report, we obtain only a single drop. But we have found that for larger system sizes multiple such drops are formed spontaneously, which are well-separated and randomly placed within the simulation box.

Figure 5 shows magnified view of the stable condensed drop shown in figure 3(D). The arrows indicate the instantaneous velocity directions of the agents. The directions appear to be random. We also find that there is no vortex-like collective motion within the drop. It is evident from figure 4 which shows that in steady state, average (over agents) angular velocity of the agents ω_z fluctuates about zero with time.

The shape of the drop is fluctuating over time. Though the time-averaged shape of the drop is circular in steady state, as is shown in figure 6. The density distribution within the drop is not uniform. It has a maximum at a certain radial distance from the center. This is clear from figure 7, where we see that the steady state density of the agents along the radius of the drop peaks at a distance about half the radial size of the drop which is about 0.25.

The most remarkable feature of this drop state is that it is stabilized by a certain amount of noise, that is to

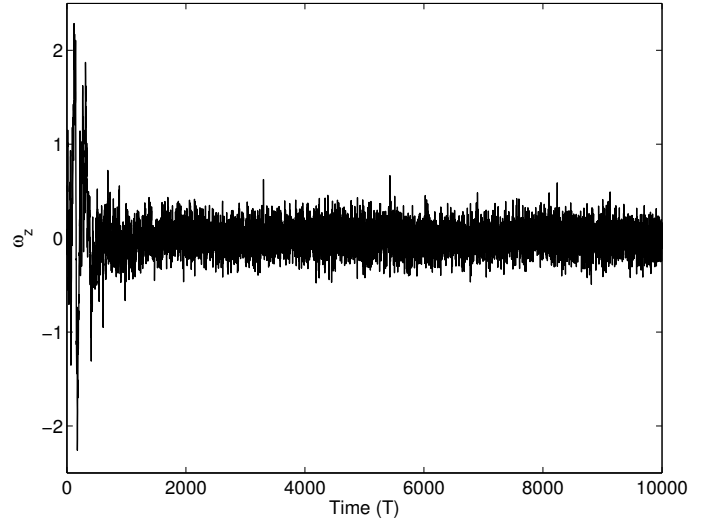


FIG. 4. Average angular velocity (ω_z) vs t plot. Parameters are $N = 576, R = 1, \rho = 1, \eta = 0.3\pi, \phi = 0.24\pi$. Position vector is with respect to the origin of the box.

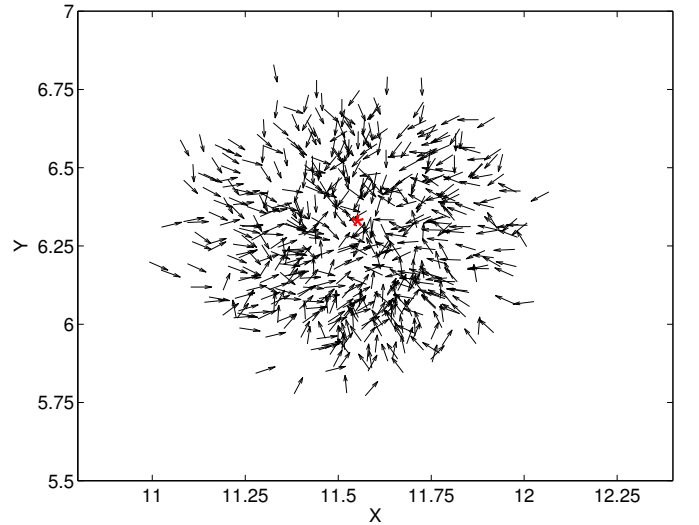


FIG. 5. Snapshot corresponding to instance (D) in figure 3. Center of mass of the system is indicated by the asterisk.

say, it is absent for zero noise. This is shown in figures 8 and 9. The figure 8 shows the order parameter ψ against view-angle ϕ for different noise levels. The dip in the order parameter (which results from the drop formation as described above) is most pronounced for a noise $\eta = 0.3\pi$, and is absent for $\eta \leq 0.20\pi$ and $\eta > 0.35\pi$. It is also apparent in figure 9, where in steady state, the time-averaged radius of gyration R_g of the system is plotted against the noise η . In this plot the drop-state of the system is reflected as a small radius of gyration (≈ 1.25), which persists over a noise range of $\eta = 0.20\pi - 0.35\pi$.

We have analyzed the dynamics of agents with following velocity autocorrelation function

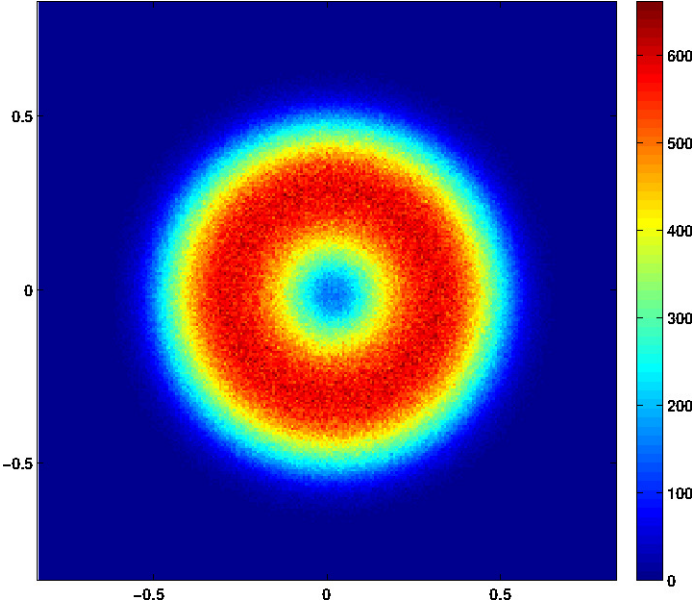


FIG. 6. The plot shows distribution of position of $N = 576$ particles within the drop for 15000 steps. The position is with respect to the center of mass which is placed at the origin in the plot. The parameters are $R = 1$, $T = 15000$ to 30000 , $\eta = 0.3\pi$, $\phi = 0.24\pi$.

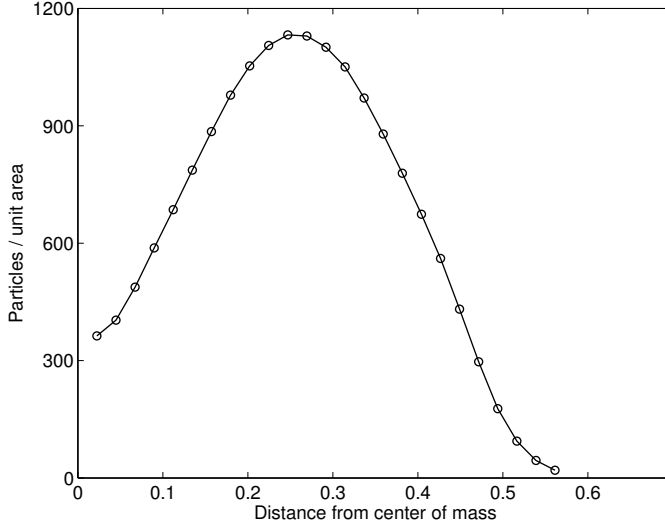


FIG. 7. Radial distribution of the particles from center of mass. The system size is $N = 576$. The distribution is calculated over the average of $T = 15000$ to 30000 .

$$\chi(t) = \left\langle \frac{v_i(t+1) \cdot v_i(t)}{|v_i(t+1)||v_i(t)|} \right\rangle \quad (13)$$

This function provides average of the cosine of the change of the angle executed by an agent between two consecutive steps, which we call ‘turn-around angle’. The angular brackets indicate averaging over agents, over time steps in steady state and over multiple realizations.

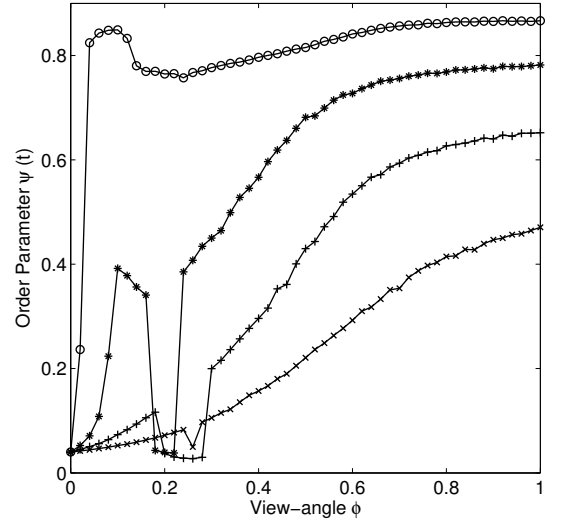


FIG. 8. Behavior of the system to the various noise values. The circle, star, plus sign and cross corresponds to noise $\eta = 0.1\pi, 0.2\pi, 0.3\pi, 0.4\pi$ respectively. Only a single configuration is used. The parameters are : $N = 576$, $R = 1$, $\rho = 1$, $\phi = 0.24\pi$.

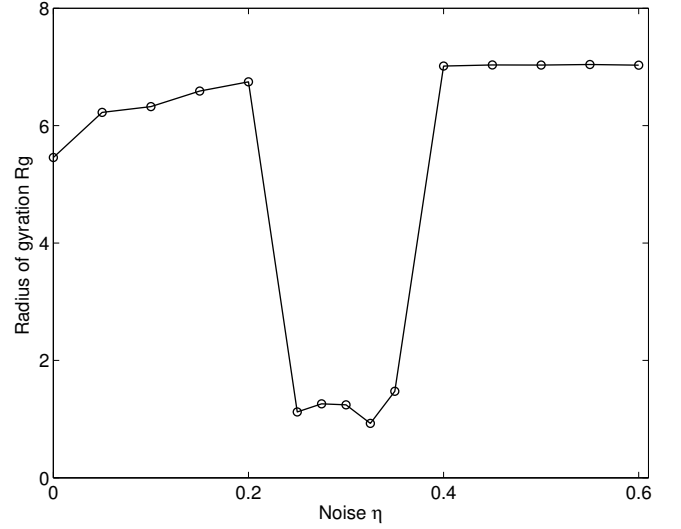


FIG. 9. Radius of gyration Rg vs noise η (in units of π) plot. The parameters are : $N = 576$, $R = 1$, $\rho = 1$, $\phi = 0.24\pi$. Each point is calculated as an average of 20 configurations and for each configuration the length of simulation was $T = 2 \times 10^4$ to $T = 10^5$.

$\chi(t)$ is plotted with varying view-angle in Fig. 10. We can see that $\chi(t)$ has a value of ≈ -0.75 in the drop state (view-angle range of 0.18π to 0.3π) and otherwise $\chi \approx +0.8$. Thus in the drop state the turnaround angle is $\approx 139^\circ$ and otherwise it is $\approx 37^\circ$. Therefore the process that spontaneously confines the agents within the drop is as follows. Each agent finds other agents in front of it within a narrow cone and obtains their average heading, which happens to be roughly opposite to its own. But

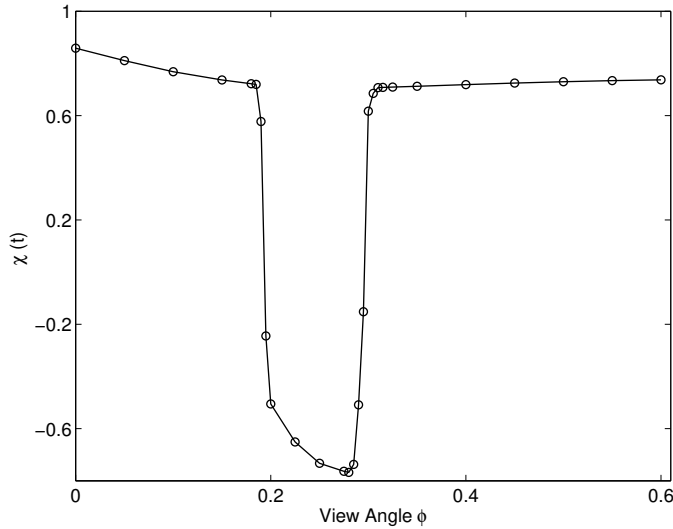


FIG. 10. Plot of average velocity autocorrelation $\chi(t)$ vs view-angle ϕ (in units of π). The parameters are system size $N = 576$, $\eta = 0.3\pi$, $R = 1$, $\rho = 1.0$. Each point is calculated as an average of 20 configurations and for each configuration the length of simulation was $T = 2 \times 10^4$ to $T = 10^5$.

it persists with its current heading for one time step due to the delay, and then it turns around by a large angle (about 139° as mentioned). Such successive large turn-arounds effectively confine the agent to small region, with a linear size approximately equal to its step-length.

VI. SUMMARY AND CONCLUSIONS

We have explored the collective behavior of self-propelled particles or agents in presence of limited (i.e.,

less than 2π radians) VC and delayed response to the motion of their neighbors. Limited VC introduces non-reciprocity in the interaction among the agents. In absence of any attractive interaction among the agents, the combination of non-reciprocity and delay are enough to produce a remarkable state where agents spontaneously condense into a dense ‘drop’. The drop remains essentially pinned in space. This happens within a particular parameter space of the opening angle of VC, density and noise level of the agents. In that parameter space the motion of the agents consists of high-angle (approximately 139°) turn-arounds, which effectively confine the agents to the drop. Within the drop, the position distribution of the agents is radially symmetric and no vortex like motion is observed. Importantly, the drop state is stabilized within a finite band of noise. With noise below than a certain level the drop becomes unstable. The drop is destabilized also at higher noise level.

VII. ACKNOWLEDGMENTS

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